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Interactions Between Branes and Matrix Theories

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We review some tests of the 0-brane and instanton matrix models based on comparing long-distance interaction potentials between branes and their bound states (with 1/2, 1/4 or 1/8 of supersymmetry) in supergravity and in super Yang-Mills descriptions. We first consider the supergravity-SYM correspondence at the level of the leading term in the interaction potential, and then describe some recent results concerning the subleading term and their implications for the structure of the 2-loop F^6 term in the SYM effective action.

1. Introduction

Below we shall review some recent results about the correspondence between type II supergravity and matrix theory (or super Yang-Mills) descriptions of long-distance interactions of certain p-branes [1–4] (see also [5,6]). We shall emphasize the common features underlying the agreement between the two pictures for different brane configurations with varied amounts of supersymmetry.

One of the ideas behind the Matrix theory proposal [7] (considered in weak-coupling limit) is that one should treat 0-branes as fundamental, effectively building other branes out of large numbers of 0-branes. That this is possible in principle follows from the existence of open string theory description of D-branes, i.e. from T-duality relating a system of D0-branes on a torus T^p to Dp-branes wrapped over the dual torus \tilde{T}^p [8,9]. Similarly, one may consider D-instantons as basic building blocks for D-branes of type IIB theory. The clusters of N D0-branes or N D-instantons may be described (at low energies) by $U(N)$ SYM theories obtained by reduction from 10 to 1 or 0 dimensions [10], which then define the corre-

sponding ‘0-brane’ [7] and ‘instanton’ [11] matrix models.

Starting with the instanton model ($S_{-1} = \frac{1}{4g_s} \text{tr}[X_a, X_b]^2 + \text{fermionic terms}$) and expanding near the vacuum $\bar{X}_0 = \text{diag}(\tilde{x}_0^{(1)}, \dots, \tilde{x}_0^{(N)})$, $\bar{X}_m = 0$, corresponding to the instantons distributed along the euclidean time direction, one may relate S_{-1} to the 0-brane matrix model action by using that in the large N limit $\bar{X}_0 \rightarrow i\partial/\partial x_0$ [11]. An example of a non-trivial classical solution ($[X_a, [X_a, X_b]] = 0$) is provided by $[\bar{X}_a, \bar{X}_b] = iF_{ab}I_{N \times N}$, or, e.g., $\bar{X}_a = i\partial_a + A_a$, $A_a = -\frac{1}{2}F_{ab}x_b$, where F_{ab} ($a, b = 0, \dots, p$) are non-vanishing constants. Such backgrounds describe non-marginal bound states $p + (p-2) + \dots + (-1)$ of type IIB Dp-branes with other branes. Similar configurations ($F_{ab} \rightarrow F_{mn}$, $m, n = 1, \dots, p$) in the 0-brane matrix model [7,12,13] describe type IIA 1/2 supersymmetric non-marginal bound states of branes $p + (p-2) + \dots + 0$. For example, a ‘D-string’ in the instanton matrix model ($p = 1$), i.e. a D-string bound to D-instantons [14], is T-dual to $2 + 0$ bound state in type IIA theory, or ‘longitudinal M2-brane’. BPS states with 1/4 of supersymmetry ($3||(-1)$ or $4||0$) may be represented by \bar{X}_a with self-dual commutator $[\bar{X}_a, \bar{X}_b]$ [15,12,13].

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Here we shall consider only branes wrapped over tori, i.e. the case of compactified matrix theory which is described by the SYM theory on the dual torus [7,9,12,16] (\tilde{T}^{p+1} in IIB case or \tilde{T}^p in IIA case). Thus instead of representing, e.g., IIA brane backgrounds in terms of large N matrices (or differential operators) of $D = 1$ SYM theory (as was done in [7,17–19,1,2]) we shall use equivalent but more straightforward representation in terms of $D = p + 1$ SYM backgrounds. The non-marginal $1/2$ supersymmetric bound states of branes are then described by constant magnetic backgrounds F_{mn} , while marginal and non-marginal $1/4$ supersymmetric bound states (e.g., $1||0$ and $4||0$) – by $1/2$ supersymmetric BPS states of SYM theory (wave $F_{m+} = 0$ and instanton $F_{mn} = F_{mn}^*$) and their superpositions with magnetic backgrounds, see, e.g., [20–23]. The $1/8$ supersymmetric bound states of branes may be described by more general $1/4$ supersymmetric SYM configurations which are superpositions of the wave and/or instanton backgrounds (see [24–28,4] and below).

In certain cases of BPS bound states of branes (having non-trivial 0-brane content in IIA case or instanton content in IIB case)² one may expect a correspondence between their description in terms of curved supersymmetric backgrounds of type II supergravity compactified on T^p and their description as SYM backgrounds on the dual torus \tilde{T}^p . On SYM side, though it may seem that one may not actually need to assume that N is large in order to have this correspondence [29], one, in fact, is to consider only planar diagrams corresponding to large N limit. As discussed in [3,4], on supergravity side, this corresponds to viewing IIA configurations of branes as resulting from a $D = 11$ theory in which a null direction $x_- = x_{11} - t$ is compactified [29]. This is formally equivalent to the prescription of computing the interaction potentials between branes by taking the 0-brane harmonic function without its standard asymptotic value 1 (see below). As we shall see, a similar $H \rightarrow H - 1$ prescription applies in

²Equivalently, the required configuration of branes should be reducible (by T-duality) to two parallel (wrapped over T^p) bound states of p-branes with other branes, e.g., $4+2$ and $4||0$, which should have SYM description.

the IIB instanton case.

On the SYM side, the interaction potential between two different BPS configurations of branes is represented by the SYM effective action Γ computed in an appropriate SYM background [30,7,11,17,18]. More precisely, the effective action in question should be obtained by integrating out only massive excitations in a background representing separated clusters of branes. However, in the case when each cluster is a BPS state with vanishing self-energy corrections one may not distinguish the full effective action from the "interaction" part of it. In general, both the vectors A_a ($a = 0, \dots, D - 1$) and the scalars X_i ($i = D, \dots, 9$) may have non-trivial background values. Consider, for example, a system of a 0-brane probe interacting with a BPS bound state of branes (wrapped over T^p), containing, in particular, N_0 0-branes. Under T-duality this becomes a system of a Dp-brane probe with charge n_0 and a Dp-brane source with charge N_0 bound to some other branes of lower dimensions (both probe and source being wrapped over \tilde{T}^p). If the probe and the source are separated by a distance r in the direction 8 and the probe has velocity v along the transverse direction 9, this configuration may be described by the following $u(N)$, $N = n_0 + N_0$, SYM background on \tilde{T}^p : $\bar{A}_a = \begin{pmatrix} 0_{n_0 \times n_0} & 0 \\ 0 & A_a \end{pmatrix}$, $\bar{X}_i = \begin{pmatrix} 0_{n_0 \times n_0} & 0 \\ 0 & X_i \end{pmatrix}$, $\bar{X}_8 = \begin{pmatrix} r I_{n_0 \times n_0} & 0 \\ 0 & 0_{N_0 \times N_0} \end{pmatrix}$, $\bar{X}_9 = \begin{pmatrix} vt I_{n_0 \times n_0} & 0 \\ 0 & 0_{N_0 \times N_0} \end{pmatrix}$, where A_a and X_i are $N_0 \times N_0$ matrices in the fundamental representation of $u(N_0)$ which describe the source bound state. The dependence on derivatives of the scalar fields X_i may be formally determined from the dependence of the effective action Γ on the gauge field in a higher-dimensional background representing T-dual ($X_i \rightarrow A_i$) configuration. In particular, the dependence on the velocity v may be described by a gauge field background $F_{09} \sim v$ [31,32,2]. This is formally the same as an extra constant field strength background switched on the probe in the euclidean D-instanton model case.

The background value of X_8 plays the role of an IR cutoff $M \sim r$ in (the interaction part of) Γ (in the open string theory picture it is related to the mass of the open string states stretched between the probe and source branes). The long-distance interaction potential \mathcal{V} will be given by the leading IR terms in the expansion of Γ in powers of $1/M$, or, equivalently, by the low-energy expansion in powers of the gauge field strength F .

On the supergravity side, the interaction potential may be determined from the action of a brane probe moving in a curved background produced by a brane source. For example, the action for a 0-brane probe in a background produced by a marginal bound state of branes $1\|0, 4\|0, 4\perp 1\|0$ or $4\perp 4\perp 4\perp 1\|0$ (which is essentially the same as the action for a $D = 11$ graviton scattering off the corresponding M-brane configuration 2+wave, 5+wave, 2 \perp 5+wave or 5 \perp 5 \perp 5+wave) has the following general structure [33,34,24]

$$I_0 = -T_0 \int dt H_0^{-1} [\sqrt{1 - H_0 H_{(1)} \dots H_{(k)} v^2} - 1] \\ \equiv \int dt [\tfrac{1}{2} T_0 v^2 - \mathcal{V}(v, r)] . \quad (1)$$

Here H_0 and $H_{(1)}, \dots, H_{(k)}$ ($k = 1$ for 1/4 supersymmetric bound states and $k = 2$ or 3 for 1/8 supersymmetric bound states) are the harmonic functions $H_{(i)} = 1 + Q_i/r^{7-p}$ representing the constituents of the bound state. Since for D-branes [8] $Q_i \sim g_s N_i$ and $T_0 \sim n_0 g_s^{-1}$ (g_s is the string coupling constant), the long-distance expansion of the classical supergravity interaction potential \mathcal{V} has the following form³

$$\mathcal{V} = \sum_{L=1}^{\infty} \mathcal{V}^{(L)} = \frac{n_0}{g_s} \sum_{L=1}^{\infty} \left(\frac{g_s}{r^{7-p}} \right)^L k_L(v, N_i) , \quad (2)$$

so that the $(1/r^{7-p})^L$ term has the same g_s dependence as in the L -loop term in SYM theory with coupling $g_{YM}^2 \sim g_s$. The detailed structure of the coefficients in (2) reflects the special role played by the 0-brane function H_0 and the

³For simplicity, here we are assuming that the bound state has only RR charges; cases with non-vanishing fundamental string charge or momentum ($Q_1 \sim g_s^2$) can be treated in a similar way, see [24,38,4] and below.

presence of the product of the ‘constituent’ harmonic functions (which is a direct consequence of the ‘harmonic function rule’ structure [35] of the supergravity backgrounds representing marginal BPS bound states of branes).

To have a precise agreement between the supergravity and SYM expressions for the potential (already at the leading level) one needs to assume that in expanding (1) in powers of $1/r^{7-p}$ one should set $H_0 = Q_0/r^{7-p}$, $Q_0 \sim g_s N_0$. This prescription may be interpreted in two possible ways (which are equivalent for the present purpose of comparing interaction potentials). One may assume (as was done in [18,19,1]) that N_0 is large for fixed r (larger than N_1, \dots, N_k and any other charge parameters that may be present in non-marginal brane configurations), so that $H_0 = 1 + Q_0/r^{7-p} \approx Q_0/r^{7-p}$. Alternatively, one may keep N_0 finite but consider the $D = 10$ brane system as resulting from an M-theory configuration with $x_- = x_{11} - t$ being compact [29]: as was pointed out in [3], the dimensional reduction of a $D = 11$ gravitational wave combined with M-brane configurations along x_- results in supergravity backgrounds with $H_0 = Q_0/r^{7-p}$.

The formal technical reason why the leading-order SYM and supergravity potentials happen to agree in certain simple cases is related to the fact that the combination of F^4 terms that appears in the 1-loop SYM effective action is also the same as the one in the expansion of the Born-Infeld action⁴ [36], but the latter is closely related to the action of a D-brane probe moving in a supergravity background. This becomes especially clear in the type IIB (instanton model) context [2], provided one takes into account that because of the T-duality involved, the relevant gauge field backgrounds which appear in the SYM and supergravity descriptions are related by $F_{ab} \rightarrow (F_{ab})^{-1}$ [2]. (equivalently, one may consider directly the T-dual system of p-brane parallel to $p + \dots + (-1)$ brane; in that case the gauge field on the brane and the SYM background are related directly).

The known (weak-coupling string theory) explanation [30,18] of the precise agreement be-

⁴This fact is due to maximal supersymmetry of SYM theory: the 1-loop F^4 terms in the bosonic YM theory are different from the combination in BI action.

tween the leading-order supergravity and 1-loop SYM potentials in certain simple cases uses the observation that for configurations of branes with sufficient amount of underlying supersymmetry, the long-distance and short-distance limits of the string-theory potential (represented by the annulus diagram [8]) are the same. That implies that the leading-order (long-distance) interaction potential determined by the classical supergravity limit of the closed string theory is the same as the (short-distance) one-loop potential produced by the massless (SYM) open string theory modes.

The results of [37,3,4] suggest that this supergravity-SYM correspondence should extend beyond the leading-order level. One may conjecture that, in general, the existence of the open string theory description of D-branes combined with enough supersymmetry implies again the agreement between long-distance and short-distance limits of higher open string loop terms in the interaction potential. Equivalently, that would mean that (i) the leading IR part of the L -loop term in the $SU(N)$ SYM effective action in $D = 1 + p$ dimensions has a universal $F^{2L+2}/M^{(7-p)L}$ structure, and (ii) computed for a SYM background representing a configuration of interacting branes, the $F^{2L+2}/M^{(7-p)L}$ term should reproduce the $1/r^{(7-p)L}$ term in the corresponding classical supergravity potential.

Below we shall first demonstrate the agreement between the leading-order terms in the SYM and supergravity expressions for the interaction potential (section 2) and then discuss what is known about that correspondence at the level of sub-leading terms [3,4] (section 3). Some concluding remarks will be made in section 4.

2. Leading-order interaction potentials

2.1. SYM effective action

In general, the effective action of the $D = p + 1$ dimensional $U(N)$ SYM theory on \tilde{T}^p ($S = -\frac{1}{2g_{\text{YM}}^2} \int d^{p+1}\tilde{x} \text{tr} F^2 + \dots$, $g_{\text{YM}}^2 = (2\pi)^{-1/2} g_s \tilde{V}_p$) for a purely gauge field background and with an

explicit IR cutoff M has the following structure⁵

$$\Gamma = \sum_{L=1}^{\infty} (g_{\text{YM}}^2 N)^{L-1} \int d^{p+1}\tilde{x} \sum_n \frac{c_{nL} F^n}{M^{2n-(p-3)L-4}}.$$

We will be interested only a special subset of terms in Γ (generalising the ‘diagonal terms’ in [3]) which have the same coupling g_s , 0-brane charge N_0 and distance $r = M$ dependence as the terms in the long-distance expansion (2) of the classical supergravity interaction potential \mathcal{V} between a Dp-brane probe (with tension $\sim n_0/g_s$) and a Dp-brane source (with charge parameter $\sim g_s N_0$). We shall assume that the SYM backgrounds describing individual branes are supersymmetric, so that the effective action vanishes when evaluated on each of them separately (i.e. its non-vanishing part will represent the interaction between branes).

One may conjecture that due to maximal underlying supersymmetry of the SYM theory, the terms $F^{2L+2}/M^{(7-p)L}$ represent, in fact, the *leading* IR (small F or, equivalently, large M) contribution to Γ at L -th loop order. This is indeed true for $L = 1$ [36] and, in view of the results of [37] (for $p = 0$) and [39] (for $p = 3$) this should be true also for $L = 2$. The sum of such leading IR terms at each loop order will be denoted as Γ . Thus

$$\Gamma = \frac{1}{2g_{\text{YM}}^2 N} \sum_{L=1}^{\infty} \int d^{p+1}\tilde{x} \left(\frac{a_p g_{\text{YM}}^2 N}{M^{7-p}} \right)^L \hat{C}_{2L+2}(F),$$

where $\hat{C}_{2L+2}(F) \sim F^{2L+2}$ and a_p are universal coefficients not depending on N or L .

At the 1-loop level, $\Gamma^{(1)} = \Gamma^{(1)} + O(\frac{1}{M^{9-p}})$, where

$$\Gamma^{(1)} = \frac{a_p}{2M^{7-p}} \int d^{p+1}\tilde{x} \hat{C}_4(F), \quad (3)$$

$$\hat{C}_4 = \text{STr } C_4 = -\frac{1}{8} \text{STr} [F^4 - \frac{1}{4}(F^2)^2]$$

$$= -\frac{1}{12} \text{Tr} (F_{ab} F_{bc} F_{cd} F_{da} + \frac{1}{2} F_{ab} F_{bc} F_{da} F_{cd})$$

⁵We will consider only in the low-energy limit of the SYM theory, i.e. will not consider the UV cutoff dependent parts in the corresponding effective actions (assuming the existence of an explicit cutoff effectively provided at weak coupling by the string theory).

$$- \frac{1}{4} F_{ab} F_{ab} F_{cd} F_{cd} - \frac{1}{8} F_{ab} F_{cd} F_{ab} F_{cd}. \quad (4)$$

Here $a_p = 2^{2-p} \pi^{-(p+1)/2} \Gamma(\frac{7-p}{2})$ and STr is the symmetrised trace in the adjoint representation (for $SU(N)$ $\text{Tr} Y^4 = 2N \text{tr} Y^4 + 6 \text{tr} Y^2 \text{tr} Y^2$, so one gets the expression containing terms with single and double traces in the fundamental representation [2]). The polynomial C_4 is the same one that appears in the expansion of the BI action,

$$\sqrt{-\det(\eta_{ab} + F_{ab})} = \sum_{n=0}^{\infty} C_{2n}(F), \quad (5)$$

$$C_0 = 1, \quad C_2 = -\frac{1}{4} F^2, \quad C_4 = -\frac{1}{8} [F^4 - \frac{1}{4} (F^2)^2],$$

$$C_6 = -\frac{1}{12} [F^6 - \frac{3}{8} F^4 F^2 + \frac{1}{32} (F^2)^3], \quad \dots,$$

where $F^2 = F_{ab} F_{ba}$, $F^k = F_{a_1 a_2} F_{a_2 a_3} \dots F_{a_k a_1}$.

2.2. Potentials from SYM theory

Let us now consider several examples of different brane configurations which admit a SYM description and compute the leading-order potentials $\mathcal{V}^{(1)}$ by substituting the corresponding gauge field backgrounds into (3). We shall assume that $\Gamma^{(1)} = \int dt \mathcal{V}^{(1)}$ in the cases involving 0-branes and that $\Gamma^{(1)} = \mathcal{V}^{(1)}$ in the D-instanton ‘interaction’ cases. In what follows we shall set $2\pi\alpha' = 1$ and assume for simplicity that the volumes of the tori take self-dual values, $V_p = \tilde{V}_p = (2\pi)^{p/2}$.

In the 1/2 supersymmetric case the basic example is the interaction of two parallel non-marginal IIB bound states $p + (p-2) + \dots + (-1)$ separated by a distance.⁶ They may be represented by the following $U(N)$, $N = n_{-1} + N_{-1}$, background on the dual torus \tilde{T}^{p+1} (n_{-1} and N_{-1} are the instanton numbers of the two branes on T^p or the numbers of Dp-branes on \tilde{T}^p):

$$X_9 = \text{diag}(r I_{n_{-1} \times n_{-1}}, 0_{N_{-1} \times N_{-1}}),$$

$$F_{ab} = \text{diag}(F_{1ab} I_{n_{-1} \times n_{-1}}, F_{2ab} I_{N_{-1} \times N_{-1}}),$$

where $F_{1,2}$ are constant parameters describing the charges of the two bound states. The $su(N)$ analogue of F_{ab} (its traceless part) may be written as

$$F_{ab} = F_{ab} J_0, \quad F \equiv F_1 - F_2,$$

⁶One may consider also interactions between orthogonally oriented branes, assuming that they are wrapped around parts of common torus.

$$J_0 = \frac{1}{n_{-1} + N_{-1}} \text{diag}(N_{-1} I_{n_{-1} \times n_{-1}}, -n_{-1} I_{N_{-1} \times N_{-1}}).$$

Since this background is abelian, STr is equivalent to Tr and thus the coefficient \hat{C}_4 in (3) determining the interaction potential is simply

$$\hat{C}_4 = -\frac{1}{4} n_{-1} N_{-1} [F^4 - \frac{1}{4} (F^2)^2]. \quad (6)$$

A particular example is that of the interaction between an instanton and a $p + (p-2) + \dots + (-1)$ state (here $F_1 = 0$) [40,2].

Similar result is found in the type IIA case, for example, for a 0-brane (with velocity v) interacting with $p + \dots + 0$ IIA bound state described by $F_{mn} = \text{diag}(0_{n_0 \times n_0}, F_{mn} I_{N_0 \times N_0})$ ($m, n = 1, \dots, p$). The IIB and IIA two cases are formally related by $F_{ab} \rightarrow (F_{09} = v, F_{mn})$, so that here [1,2]

$$\hat{C}_4 = -\frac{1}{4} n_0 N_0 [F^4 - \frac{1}{4} (F^2)^2 - v^2 F^2 + v^4]. \quad (7)$$

Special cases, e.g., 0-brane - 0-brane ($F = 0$) and 0-brane - (2+0)-brane ($F_{12} = \frac{n_2}{n_0}$) interactions were discussed in [17,32,18,19,41].

An example of a bound state with 1/4 of supersymmetry is $4||0$ which may be described [15] by a self-dual $SU(N)$ background on \tilde{T}^4 :

$$F_{mn} = F_{mn}^*, \quad \int d^4 \tilde{x} \text{tr}(F_{mn} F_{mn}) = (4\pi)^2 N_4,$$

or, explicitly, by a constant background:

$$F_{12} = F_{34} = q J_1, \quad q^2 = \frac{N_4}{N_0},$$

$$J_1 \equiv \text{diag}(0_{n_0 \times n_0}, I_{\frac{1}{2} N_0 \times \frac{1}{2} N_0}, -I_{\frac{1}{2} N_0 \times \frac{1}{2} N_0}).$$

Since the resulting background is commuting, the potential is again given essentially by (6) or (7). For example, in the case of the $(4||0) - (4||0)$ system of two parallel $4||0$ states with charges (n_4, n_0) and (N_4, N_0) we find [1]

$$\mathcal{V}^{(1)} = -\frac{n_0 N_0}{16r^3} [4(\frac{N_4}{N_0} + \frac{n_4}{n_0}) v^2 + v^4]. \quad (8)$$

Similarly, for the static potential between orthogonally oriented (within 6-torus) $2 + 0$ and $4||0$ states we get [1]

$$\mathcal{V}^{(1)} = -\frac{n_0 N_0}{16r} [(\frac{n_2}{n_0})^4 - 4(\frac{n_2}{n_0})^2 \frac{N_4}{N_0} + O(v^2)].$$

Analogous expressions are found in the case of $1||0$ bound state of a fundamental string and a 0-brane which is describes by a plane wave background $X_2 = X_2(\tilde{x}_1 + t)$ or, in the T-dual picture,

$A_2 = A_2(\tilde{x}_1 + t)$, $F_{12} = F_{02} = hI_{N_0 \times N_0}$, where $h = h(\tilde{x}_1 + t)$ is a periodic function normalised so that $\langle h^2 \rangle = g_s \frac{N_1}{N_0}$, where N_1 is the string winding number (see [38,4]). for example, for a 0-brane interacting with $4||0$ we get the expression similar to the $0-(4||0)$ case (i.e. (8) with $n_4 = 0$)

$$\mathcal{V}^{(1)} = -\frac{n_0 N_0}{8r^6} (4 \langle h^2 \rangle v^2 + v^4) .$$

Similar expressions describe also interactions involving the corresponding T-dual type IIB bound states $3||(-1)$ and $(-1)+\text{wave}$.

To determine the leading-order potentials for configurations involving $1/8$ supersymmetric states one needs to find their SYM description and substitute the resulting backgrounds into (3). The configuration of a 0-brane interacting with $4\perp 1||0$ state wrapped over T^5 (corresponding to extremal $D = 5$ black hole) may be described by a combination of an instanton and a momentum wave (carried, in general, by vectors *and* scalars), or, explicitly (after T-duality trading scalar backgrounds for the vector ones) [4]⁷

$$F_{09} = vJ_0 , \quad F_{12} = F_{34} = qJ_1 , \quad (9)$$

$$F_{51} = F_{01} = hJ_0 , \quad F_{56} = F_{06} = wJ_0 , \quad (10)$$

where the $su(n_0 + N_0)$ matrices J_0, J_1 were defined above, $q^2 = \frac{N_4}{N_0}$, and the periodic ‘vector wave’ and ‘scalar wave’ functions $h = h(\tilde{x}_5 + t)$ and $w = w(\tilde{x}_5 + t)$ satisfy

$$\langle h^2 + w^2 \rangle = \frac{1}{L^5} \int d\tilde{x}_5 (h^2 + w^2) = g_s \frac{N_1}{N_0} ,$$

i.e. $\langle h^2 + w^2 \rangle$ is proportional to the total momentum of the wave in the T-dual $(5||1+\text{wave})$ configuration. The \hat{C}_4 -coefficient of the corresponding leading-order potential is found to be (equivalent results were obtained in [24,28])

$$\hat{C}_4 = -\frac{1}{4} n_0 N_0 [4v^2 q^2 + 4v^2 (h^2 + w^2) + v^4] . \quad (11)$$

The same expression is found for the 0-brane interaction with $4\perp 4\perp 4||0$ bound state wrapped

⁷This ‘instanton+wave’ $u(N)$ gauge field background, which should be representing the marginal BPS $5||1+\text{wave}$ configuration invariant under $1/8$ of $\mathcal{N} = 2, D = 10$ type IIB supersymmetry, is, indeed, preserving $1/4$ of the $\mathcal{N} = 1, D = 10$ supersymmetry of the SYM theory [4].

over T^6 (corresponding to extremal $D = 4$ black hole), or for $6-(6||2\perp 2\perp 2)$ interaction on \tilde{T}^6 . The $6||2\perp 2\perp 2$ configuration may be described by an ‘overlap’ of the three 4d instantons on \tilde{T}^6 , i.e. by the following $su(N_0)$ constant gauge field strength background [4]

$$F_{14} = F_{23} = q_1 \lambda_1 , \quad F_{45} = F_{36} = q_2 \lambda_2 , \quad (12)$$

$$F_{15} = -F_{26} = q_3 \lambda_3 ,$$

where $q_k^2 = \frac{N_{4(k)}}{N_0}$ and λ_k are some three independent $su(N_0)$ matrices normalised so that this gauge configuration produces the right 2-brane charges (and only them) on 6-brane. One possible choice of λ_k is the following ‘*commuting*’ one (assuming that N_0 is a multiple of 4): $\lambda_k = \mu_k \otimes I_{\frac{N_0}{4} \times \frac{N_0}{4}}$, where μ_k are the diagonal 4×4 matrices (used in [23]) $\mu_1 = \text{diag}(1, 1, -1, -1)$, $\mu_2 = \text{diag}(1, -1, -1, 1)$, $\mu_3 = \text{diag}(1, -1, 1, -1)$. A ‘*non-commuting*’ choice is to set λ_k to be proportional to the Pauli matrices σ_k , i.e. $\lambda_k = \sigma_k \otimes I_{\frac{N_0}{2} \times \frac{N_0}{2}}$. Both choices represent $1/4$ supersymmetric configurations in the $D = 6 + 1$ SYM theory [4]. The leading-order potential in this case is proportional to

$$\hat{C}_4 = -\frac{1}{4} n_0 N_0 [4v^2 (q_1^2 + q_2^2 + q_3^2) + v^4] . \quad (13)$$

Comparing (11),(13) with various special cases discussed above we conclude that, as might be expected, the *leading-order* SYM interaction potentials for *marginal* bound states are essentially the sums of pair-wise interactions between constituent branes. The same will, of course, be true on the supergravity side (cf. (1)), and the potentials will be in full agreement.

It should be mentioned also that the leading-order interactions involving non-supersymmetric bound states of branes [23,42,43] or near-extremal black holes [28] are again described by the universal F^4 action (3).

2.3. Potentials from supergravity

To find the supergravity potentials we shall use the probe method, i.e. consider the action of a D-brane probe ($I_p = -T_p [\int d^{p+1} x e^{-\phi} \sqrt{\det(G_{ab} + G_{ij} \partial_a X^i \partial_b X^j + \mathcal{F}_{ab})} - \sum C e^{\mathcal{F}}]$) in a curved background produced by a brane bound

state as a source. The key example in the 1/2 supersymmetric brane case is the interaction of a D-instanton with a type IIB non-marginal bound state $p + (p - 2) + \dots + (-1)$. The action for the latter considered as a probe may be found by switching on a constant \mathcal{F}_{ab} background on the Dp-brane world volume [44]. The fluxes produced by \mathcal{F}_{ab} determine the numbers of branes [15] of each type in the bound state. In particular, the instanton number is

$$n_{-1} = \frac{n_p V_{p+1}}{(2\pi)^{(p+1)/2}} \sqrt{\det \mathcal{F}_{ab}} = n_p \sqrt{\det \mathcal{F}_{ab}}$$

(we assume that $T_p = n_p g_s^{-1} (2\pi)^{(1-p)/2}$ and $V_{p+1} = (2\pi)^{(p+1)/2}$). Substituting the D-instanton background [45] ($ds_{10}^2 = H^{1/2}(dx_a dx_a + dx_i dx_i)$, etc.) ‘smeared’ along the directions of T^{p+1} into the Dp-brane action and ignoring the dependence of X_i on the world-volume coordinates x_a we find

$$I_p = -T_p V_{p+1} H^{-1} \times [\sqrt{\det(H^{1/2} \delta_{ab} + \mathcal{F}_{ab})} - \sqrt{\det \mathcal{F}_{ab}}] \\ = -T_{-1} H^{-1} [\sqrt{\det(\delta_{ab} + H^{1/2} \mathcal{F}_{ab}^{-1})} - 1],$$

where $T_{-1} = 2\pi g_s^{-1} n_{-1} = T_p V_{p+1} \sqrt{\det \mathcal{F}_{ab}}$. Thus we got the BI action with the field $\mathcal{F}_{ab} \rightarrow \mathcal{F}_{ab}^{-1}$ in a curved background.

To establish the agreement with the leading-order potential in the SYM theory one may assume that \mathcal{F}_{ab} (and thus n_{-1}) is very large and expand in powers of \mathcal{F}^{-1} [1], or, alternatively, drop 1 in the source D-instanton harmonic function H , taking it as $H = Q_{-1}/r^{7-p}$, $Q_{-1} \sim N_{-1} g_s$. This prescription may be justified by the assumption that N_{-1} is large.⁸

⁸By analogy with a similar prescription in the type IIA (0-brane) case [3], it may also be given the following heuristic interpretation. As was noted in [47], the D-instanton solution of [45] is formally a reduction of a gravitational plane wave from 12 to 10 dimensions, $ds_{12}^2 = dx_+ dx_- + K(x) dx_- dx_- + dx_a dx_a$, $x_{\pm} = x_{12} \pm x_{11}$, $a = 0, 1, \dots, 9$, $K = Q/r^8$. Reducing along x_{11} and x_{12} , i.e. $ds_{12}^2 = -e^{-\phi} dx_{11}^2 + e^{\phi} (dx_{12} + C_0 dx_{11})^2 dx_a dx_a$, one finds the instanton background $e^{\phi} = H = 1 + K$, $C_0 = H^{-1} - 1$ with the string-frame metric $ds_{10}^2 = H^{1/2} dx_a dx_a$. If instead one reduces along x_- , x_+ one finds $e^{\phi} = K$, $C_0 = K^{-1}$, $ds_{10}^2 = K^{1/2} dx_a dx_a$, i.e. the background with $H \rightarrow H - 1$ (see also [46] for a discussion

Since the F^4 term in the expansion of the BI action (5) is given by the C_4 combination, the resulting leading term in the interaction potential is found to have the same structure and the coefficient as in (3),(6), i.e. [2]

$$\mathcal{V}^{-1} = -\frac{a_p \tilde{V}_{p+1}}{8r^{7-p}} n_{-1} N_{-1} [F^4 - \frac{1}{4}(F^2)^2],$$

where $F_{ab} \equiv \mathcal{F}_{ab}^{-1}$. The fact that the two abelian field strengths appearing in the supergravity and the SYM descriptions are related by the *inversion* is a consequence of T-duality. T-duality transforms the $(-1) - (p + \dots + (-1))$ system on T^p into a system of ‘pure’ Dp-brane and Dp-brane with extra charges $p - ((-1) + \dots + p)$ on \tilde{T}^p which is expected to have the $U(n_{-1} + N_{-1})$ SYM theory description.

Closely related expressions are found in the type IIA case of 0-brane interacting with $p + \dots + 0$ non-marginal bound state. The $(p + \dots + 0)$ probe action in the 0-brane background is

$$I_p = -T_p V_p \int dt H_0^{-1} \times [\sqrt{(1 - H_0 v^2) \det(H_0^{1/2} \delta_{mn} + \mathcal{F}_{mn})} - \sqrt{\det \mathcal{F}_{mn}}],$$

where \mathcal{F}_{mn} describes other brane charges, e.g., $n_0 = n_p (2\pi)^{-p/2} V_p \sqrt{\det \mathcal{F}_{mn}}$. This action may be rewritten as

$$I_p = -T_0 \int dt H_0^{-1} \times [\sqrt{1 - H_0 v^2} \sqrt{\det(\delta_{mn} + H_0^{1/2} \mathcal{F}_{mn})} - 1], \quad (14)$$

where $T_0 = n_0 g_s^{-1} (2\pi)^{1/2}$ and $\mathcal{F}_{mn} \equiv (\mathcal{F}_{mn})^{-1}$. In this form it corresponds to a T-dual configuration, i.e. to the interaction of a p -brane source (with charge N_0) with parallel $(0 + \dots + p)$ -brane probe (with 0-brane charge n_0) moving in a relative transverse direction. Introducing the velocity

of such shifts in harmonic functions in connection with T-duality). Since $H \approx K$ at small r this background may be interpreted as a short-distance limit of the D-instanton solution. Given that the latter represents a wormhole [45], this new string-frame metric is flat everywhere, $ds_{10}^2 = Q^{1/2} r^{-4} dx_a dx_a = Q^{1/2} (d\rho^2 + \rho^2 d\Omega_9^2)$, $\rho = 1/r$.

component $F_{09} = v$ we can put this action in the same BI form as in the above type IIB example,

$$I_p = -T_0 \int dt H_0^{-1} [\sqrt{-\det(\eta_{ab} + H_0^{1/2} F_{ab})} - 1]$$

were again $H_0 = Q_0/r^{7-p}$ so that the agreement between the leading-order long-distance interaction potential and the SYM result (7) is manifest.

Next, one may consider a $p + \dots + 0$ or $p + \dots + (-1)$ brane probe described by a Dp-brane action with a constant \mathcal{F}_{mn} field strength moving in type IIA or type IIB supergravity backgrounds produced by a 1/4 or 1/8 supersymmetric marginal (or non-marginal) bound state of branes. Since the latter are known explicitly (see, e.g., [34]) the computation of the interaction potentials is straightforward. For example, the potentials in the case of 0-brane interactions with 1/4 or 1/8 supersymmetric marginal bound states have the universal form of (1). In the $0 - (4||0)$ case $I_0 = -T_0 \int dt H_0^{-1} (\sqrt{1 - H_0 H_4 v^2} - 1)$, where $H_0 = Q_0/r^3$, $H_4 = 1 + Q_4/r^3$, $Q_4 \sim N_4 g_s$, so that

$$\begin{aligned} \mathcal{V}^{(1)} &= -\frac{T_0}{8r^3} (4Q_4 v^2 + Q_0 v^4) \\ &= -\frac{n_0}{16r^3} (4v^2 N_4 + v^4 N_0), \end{aligned}$$

which is equivalent to the SYM result (8) [1]. Had we kept the constant 1 in H_0 we would get $N_0 + 2N_4$ as the coefficient of the v^4 term and would need to assume that $N_0 \gg N_4$ to get the agreement with the SYM result.

Similarly, using the explicit form of the $4\perp 4\perp 4||0$ background [48] one finds that the $0 - (4\perp 4\perp 4||0)$ interaction is described by $I_0 = -T_0 \int dt H_0^{-1} (\sqrt{1 - H_0 H_{4(1)} H_{4(2)} H_{4(3)} v^2} - 1)$, where $H_{4(k)} = 1 + Q_{4(k)}/r$, so that

$$\mathcal{V}^{(1)} = -\frac{n_0}{16r} [4v^2 (N_{4(1)} + N_{4(2)} + N_{4(3)}) + v^4 N_0],$$

which is again in agreement with the SYM expression (13).

To summarize, the SYM–supergravity correspondence observed on the above examples is formally due to (i) the BI-type structure of the actions of the non-marginal bound state branes, (ii)

the ‘product of harmonic functions’ structure of the actions in the marginal bound state case (implying additive dependence of the leading-order potential on constituent charges), and (iii) a combination of these two features in more general cases of interactions with non-marginal 1/4 or 1/8 supersymmetric bound states.

3. Subleading term in interaction potentials

The result of [3] may be interpreted as implying that the subleading term $\mathcal{V}^{(2)} \sim n_0 N_0^2 g_s v^6 / r^{14}$ in the 0-brane - 0-brane interaction potential (2) in $I_0 = -T_0 \int dt H_0^{-1} (\sqrt{1 - H_0 v^2} - 1)$ is reproduced by the leading 2-loop term in the $D = 1$ SYM effective action Γ defined in section 2.1.⁹ This is easy to check by assuming that the 2-loop coefficient \hat{C}_6 in Γ has the same structure as the 1-loop one \hat{C}_4 , i.e. is given by the (symmetrized) trace in the adjoint representation of the F^6 term appearing in the expansion of the BI action (5),

$$\hat{C}_6 = \text{STr } C_6(F). \quad (15)$$

Indeed, interpreting the velocity as an electric field component in $D = 2$ SYM theory and substituting $F_{09} = v J_0$ into (15), using that $\text{Tr } J^{2n} = 2n_0 N_0$ and separating the $n_0 N_0^2$ -term as required [3] to match the supergravity result (obtained by the probe method) one finds the precise agreement with the supergravity potential.

In [4] we attempted to test the ansatz (15) by studying the subleading terms in the potentials in more complicated examples of 0-brane interacting with bound states of branes wrapped over tori. Since an explicit computation of the 2-loop term in Γ for arbitrary non-abelian gauge field in a higher-dimensional SYM looks as a complicated problem, we followed an indirect route: making a plausible ansatz for the 2-loop term in Γ and then trying to compare it with the supergravity expressions for $\mathcal{V}^{(2)}$ on different examples with varied amount of supersymmetry, *assuming* that the supergravity-SYM correspondence should continue to hold beyond the leading order

⁹Similar SYM interpretation should apply to the discussion in [49].

as it does in the simplest 0-brane scattering case. The consistency of the resulting picture supports the basic assumption.

The first non-trivial example is the 0-brane interaction with 1/2 supersymmetric non-marginal bound state ($p + \dots + 0$). Since the action (14) has the BI structure, the subleading term in its potential part (the one which is quadratic in $H_0 = Q_0/r^{7-p}$) has the $C_6 \sim F^6$ form. Plugging the corresponding SYM background $F_{09} = vJ_0$, $F_{mn} = F_{mn}J_0$ into (15) we find the precise agreement between the SYM and the supergravity expressions for $\mathcal{V}^{(2)}$ for *arbitrary* \mathcal{F}_{mn} .

This, in fact, may be considered as a motivation for choosing (15) in the first place. A test then comes from the cases of 0-brane interaction with 1/4 supersymmetric 1||0 and 4||0 bound states. Though the supergravity action $I_0 = -T_0 \int dt H_0^{-1} (\sqrt{1 - H_0 H_1 v^2} - 1)$ and thus $\mathcal{V}^{(2)} = -\frac{1}{16r^{12}} Q_0 (4Q_1 v^4 + Q_0 v^6)$ in the 0 - (1||0) case have different structure than (14), $\mathcal{V}^{(2)}$ is still reproduced [4] by the 2-loop term in Γ after one substitutes the relevant $SU(n_0 + N_0)$ background $F_{09} = vJ_0$, $F_{12} = F_{02} = h(\tilde{x}_1 + x_0)J_0$ into \hat{C}_6 given by (15).

In the 0 - (4||0) case the supergravity potential has the same form as in the 0 - (1||0) case, but the corresponding SYM background (9) is now more complicated: it is parametrised by two independent commuting matrices J_0 and J_1 . Substituting $F_{09} = vJ_0$, $F_{12} = F_{34} = qJ_1$ into (15) gives $\text{Tr} C_6 = -\frac{1}{8} n_0 N_0 (2v^4 q^2 + v^6)$ instead of

$$\hat{C}_6 = -\frac{1}{8} n_0 N_0 (4v^4 q^2 + v^6)$$

which is needed for agreement with the supergravity potential.

It is natural to try to modify the ansatz (15) in order to correct the factor of 2 discrepancy in the v^4 term, without changing, however, the result for \hat{C}_6 in all the previous cases (which were represented by more primitive gauge field backgrounds depending on a single $SU(n_0 + N_0)$ matrix J_0). Remarkably, it turns out [4] that there is a *unique* way of achieving that goal (up to terms involving commutators of F which are discussed below and which vanish on the backgrounds we considered so far): one is to keep the same Lorentz-index

structure of the F^6 terms as in C_6 but should replace the internal index trace $\widehat{\text{STr}}$ by a different combination of tr , $(\text{tr})^2$ and $(\text{tr})^3$ terms (all such fundamental trace structures may in general appear at 2 loops)¹⁰

$$\begin{aligned} \widehat{\text{STr}}(Y_{s_1} \dots Y_{s_6}) &\equiv 2N \text{tr}[Y_{(s_1} \dots Y_{s_6)}] \\ &+ \alpha_1 \text{tr}[Y_{(s_1} \dots Y_{s_4}]\text{tr}[Y_{s_5} Y_{s_6})] \\ &+ \alpha_2 \text{tr}[Y_{(s_1} \dots Y_{s_3}]\text{tr}[Y_{s_4} \dots Y_{s_6})] \\ &+ \alpha_3 N^{-1} \text{tr}[Y_{(s_1} Y_{s_2}]\text{tr}[Y_{s_3} Y_{s_4}]\text{tr}[Y_{s_5} Y_{s_6})]. \end{aligned}$$

Y_s are the $SU(N)$ generators and $\alpha_1 = 30$, $\alpha_2 = -20$, $\alpha_3 = 0$ in the case when $\widehat{\text{STr}} Y^6 = \text{STr} Y^6$, but we need to choose $\alpha_1 = 60$, $\alpha_2 = -50$, $\alpha_3 = -30$ in order for the modified ansatz

$$\hat{C}_6 = \widehat{\text{STr}} C_6(F), \quad (16)$$

to reproduce the supergravity expressions in all of the above examples, including the 0 - (4||0) one (which is the only case among them where $\widehat{\text{STr}} C_6(F) \neq \text{STr} C_6(F)$). Indeed, one finds that for the gauge field background representing the 0 - (4||0) configuration

$$\hat{C}_6 = \widehat{\text{STr}} C_6 = -\frac{1}{8(n_0 + N_0)} n_0 N_0$$

$$\times [(n_0 + N_0)(2v^4 q^2 + v^6) + 2N_0 v^4 q^2],$$

so that the relevant $n_0 N_0^2$ term in the 2-loop coefficient $(n_0 + N_0) \hat{C}_6$ in Γ is now in agreement with the supergravity expression for the subleading potential $\mathcal{V}^{(2)}$.

A test of the consistency of (16) is provided by further examples of 0-brane interactions with 1/8 supersymmetric bound states. In the 0 - (4⊥1||0) case the subleading term in the supergravity potential (1),(2) is

$$\mathcal{V}^{(2)} = -\frac{T_0}{16r^4} [8v^2 Q_4 Q_1 + 4v^4 (Q_4 + Q_1) Q_0 + v^6 Q_0^2],$$

where $Q_1 = g_s Q_0 \frac{N_1}{N_0}$. The same expression should be reproduced by the 2-loop SYM effective action

¹⁰Since here we consider only commuting F_{ab} backgrounds, the symmetrisation does not play any role. It is, however, useful in order to isolate the terms that do not vanish in the abelian limit from additional commutator terms (see below).

Γ with the coefficient \hat{C}_6 in (16) computed for the background (10). One finds that

$$\hat{C}_6 = -\frac{1}{8(n_0 + N_0)} n_0 N_0^2 [16v^2 w^2 q^2 + 4v^4(q^2 + h^2 + w^2) + v^6] + O(n_0^2),$$

so that the v^4 and v^6 terms are indeed the same as in $\mathcal{V}^{(2)}$ for any distribution of the total momentum between the vector and scalar oscillations (representing the momentum wave along the D-string bound to D5-brane in the T-dual configuration 5||1+wave). However, the coefficient of the v^2 term is *non-vanishing* (cf. [24]) only if $w \neq 0$, i.e. only if the scalar background is excited. The v^2 term has the required coefficient provided we assume that the momentum is distributed equally between the scalar and the vector waves, i.e. if $\langle h^2 \rangle = \langle w^2 \rangle = \frac{1}{2} g_s \frac{N_1}{N_0}$.

Finally, in the $0-(4\perp 4\perp 4||0)$ case the v^4 and v^6 terms in the corresponding supergravity potential

$$\mathcal{V}^{(2)} = -\frac{n_0 g_s}{64(2\pi)^{1/2} r^2} \times \left[\frac{1}{2} v^2 (N_{4(1)} N_{4(2)} + N_{4(1)} N_{4(3)} + N_{4(2)} N_{4(3)}) + 4v^4 (N_{4(1)} + N_{4(2)} + N_{4(3)}) N_0 + v^6 N_0^2 \right], \quad (17)$$

are correctly reproduced by the \hat{C}_6 in (16) evaluated on the background (12) supplemented by the velocity component $F_{09} = vJ_0$. However, the coefficient of the v^2 term in the resulting expression for $\Gamma^{(2)}$ is vanishing for both commuting and non-commuting choice of the matrices λ_k in (12).

One should note, however, that up to this point we were ignoring the terms with commutators of F_{ab} which may, of course, be present in the 2-loop effective action, but which were vanishing on the commuting backgrounds we were discussing above. In general, one should expect, therefore, that¹¹

$$\hat{C}_6 = \widehat{\text{STr}} C_6(F) + \mathcal{C}_6, \quad \mathcal{C}_6 = O(F^4[F, F]). \quad (18)$$

¹¹Though this does not seem to be directly related to the present discussion, let us note that, in general, the F^6 -part of the *tree-level* (disc) open string theory effective action should also contain certain commutator terms in addition to the symmetrised trace terms $\text{Str} C_6(F)$ in the non-abelian BI action as defined in [14]. That such commutator terms may be needed in D-brane action applications is implied by the results of [50–52].

To demonstrate that the commutator terms \mathcal{C}_6 can, indeed, produce the needed v^2 term, let us consider, e.g.,

$$\mathcal{C}_6 \sim \text{Tr}(F_{ab} F_{ab} [F_{cd}, F_{ef}] F_{cd} F_{ef}).$$

Making the non-commutative choice of the background in (12), i.e. taking λ_k to be proportional to the Pauli matrices, one finds [4] that \mathcal{C}_6 (multiplied by $N = n_0 + N_0$ which is its coefficient in Γ) contains indeed the same v^2 contribution as in (17), i.e. the one proportional to $n_0 N_0^2 v^2 (N_{4(1)} N_{4(2)} + N_{4(1)} N_{4(3)} + N_{4(2)} N_{4(3)})$.

4. Concluding remarks

As we have discussed above, the supergravity-SYM (matrix theory) correspondence is manifest for the leading term in the long-distance interaction potential between appropriate configurations of branes in $D = 10$ (having large 0-brane number N_0 or finite N_0 but obtained from $D = 11$ using ‘null’ reduction).

We have suggested that this correspondence holds also for the subleading terms in the long-distance potential between extended branes, i.e. not only for the $D = 1$ SYM (0-brane scattering) case considered in [3]. It would be important to perform a string-theory computation of the subleading (2-loop) terms in the interaction potential, checking that the $r \rightarrow 0$ and $r \rightarrow \infty$ limits of the string result continue to agree (for relevant configurations of branes) beyond the leading 1-loop level considered in [31,30,18]. This would provide an explanation for the supergravity-SYM correspondence at the subleading level observed in [3,4].

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REFERENCES

1. I. Chepelev and A.A. Tseytlin, *Long-distance interactions of D-brane bound states and longitudinal 5-brane in M(atrix) theory*, Phys. Rev. D56 (1997) 3672, hep-th/9704127.
2. I. Chepelev and A.A. Tseytlin, *Interactions of type IIB D-branes from D-instanton matrix model*, hep-th/9705120.
3. K. Becker, M. Becker, J. Polchinski, and A. Tseytlin, *Higher order graviton scattering in M(atrix) theory*, Phys. Rev. D56 (1997) 3174, hep-th/9706072.
4. I. Chepelev and A.A. Tseytlin, *Long-distance interactions of branes: correspondence between supergravity and super Yang-Mills descriptions*, hep-th/9709087.
5. K. Becker, *Testing M(atrix) Theory at Two Loops*, Contribution to Strings '97, hep-th/9709038.
6. J. Maldacena, *Branes probing Black Holes*, Contribution to Strings '97, hep-th/9709099.
7. T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M Theory as a matrix model: a conjecture*, Phys. Rev. D55 (1997) 5112, hep-th/9610043.
8. J. Polchinski, *Dirichlet branes and Ramond-Ramond charges*, Phys. Rev. Lett. 75 (1995) 4724; *TASI Lectures on D-Branes*, hep-th/9611050.
9. W. Taylor, *D-brane field theory on compact spaces*, Phys. Lett. B394 (1997) 283, hep-th/9611042.
10. E. Witten, *Bound states of strings and p-branes*, Nucl. Phys. B460 (1995) 335, hep-th/9510135.
11. N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A Large- N Reduced Model as Superstring*, Nucl. Phys. B498 (1997) 467, hep-th/9612115.
12. O.J. Ganor, S. Ramgoolam and W. Taylor, *Branes, Fluxes and Duality in M(atrix) Theory*, hep-th/9611202.
13. T. Banks, N. Seiberg and S. Shenker, *Branes from Matrices*, hep-th/9612157.
14. A.A. Tseytlin, *On non-abelian generalisation of Born-Infeld action in string theory*, hep-th/9701125.
15. M.R. Douglas, *Branes within Branes*, hep-th/9512077.
16. L. Susskind, *T Duality in M(atrix) Theory and S Duality in Field Theory*, hep-th/9611164.
17. O. Aharony and M. Berkooz, *Membrane Dynamics in M(atrix) Theory*, Nucl. Phys. B491 (1997) 184, hep-th/9611215.
18. G. Lifschytz and S.D. Mathur, *Supersymmetry and Membrane Interactions in M(atrix) Theory*, hep-th/9612087.
19. G. Lifschytz, *Four-Brane and Six-Brane Interactions in M(atrix) Theory*, hep-th/9612223.
20. Z. Guralnik and S. Ramgoolam, *Torons and D-brane bound states*, Nucl. Phys. B499 (1997) 241, hep-th/9702099.
21. R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix String Theory*, hep-th/9703030.
22. R. Gopakumar, *BPS states in Matrix Strings*, hep-th/9704030. ep-th/9612157.
23. W. Taylor, *Adhering 0-branes to 6-branes and 8-branes*, hep-th/9705116.
24. M. Douglas, J. Polchinski and A. Strominger, *Probing Five-Dimensional Black Holes with D-Branes*, hep-th/9703031.
25. M. Li and E. Martinec, *Matrix black holes*, hep-th/9703211; *On the entropy of matrix black holes*, hep-th/9704134.
26. E. Halyo, *M(atrix) black holes in five dimensions*, hep-th/9705107.
27. R. Dijkgraaf, E. Verlinde and H. Verlinde, *5-D Black holes and matrix strings*, hep-th/9704018.
28. J. Maldacena, *Probing near extremal black holes with D-branes*, hep-th/9705053.
29. L. Susskind, *Another Conjecture about M(atrix) Theory*, hep-th/9704080.
30. M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, *D-branes and short distances in string theory*, Nucl. Phys. B485 (1997) 85, hep-th/9608024.
31. C. Bachas, *D-Brane Dynamics*, Phys. Lett. B374 (1996) 37, hep-th/9511043.
32. D. Berenstein and R. Corrado, *M(atrix) theory in various dimensions*, hep-th/9702108.
33. A.A. Tseytlin, *No-force condition and BPS combinations of p-branes in 11 and 10 dimensions*, Nucl. Phys. B487 (1997) 141, hep-th/9609212.

34. A.A. Tseytlin, *Composite BPS configurations of p-branes in 10 and 11 dimensions*, Class. Quant. Grav. 14 (1997) 2085, hep-th/9702163.
35. A.A. Tseytlin, *Harmonic superpositions of M-branes*, Nucl. Phys. B475 (1996) 149, hep-th/9604035.
36. R.R. Metsaev and A.A. Tseytlin, *On loop corrections to string theory effective actions*, Nucl. Phys. B298 (1988) 109.
37. K. Becker and M. Becker, *A two loop test of M(atr ix) theory*, hep-th/9705091.
38. R. Gopakumar and S. Ramgoolam, *Scattering of zero branes off elementary strings in matrix theory*, hep-th/9708022.
39. M. Dine and N. Seiberg, *Comments on Higher Derivative Operators in Some SUSY Field Theories*, hep-th/9705057.
40. I. Chepelev, Y. Makeenko and K. Zarembo, *Properties of D-Branes in Matrix Model of IIB Superstring*, hep-th/9701151.
41. V. Balasubramanian and F. Larsen, *Relativistic brane scattering*, hep-th/9703039.
42. J.M. Pierre, *Comparing D-branes and black holes with 0- and 6-brane charge*, hep-th/9707102; *Interactions of eight-branes in string theory and M(atr ix) theory*, hep-th/9705110.
43. E. Keski-Vakkuri and P. Kraus, *Notes on Branes in Matrix Theory*, hep-th/9706196.
44. M.B. Green and M. Gutperle, *Light-cone supersymmetry and D-branes*, Nucl. Phys. B476 (1996) 484, hep-th/9604091.
45. G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and seven-branes in type IIB superstring theory*, Phys. Lett. B370 (1996) 37, hep-th/9511080.
46. H.J. Boonstra, B. Peeters and K. Skenderis, *Duality and asymptotic geometries*, hep-th/9706192.
47. A.A. Tseytlin, *Type IIB instanton as a wave in twelve dimensions*, Phys. Rev. Lett. 78 (1997) 1864, hep-th/9612164.
48. I.R. Klebanov and A.A. Tseytlin, *Intersecting M-branes as four-dimensional black holes*, Nucl. Phys. B475 (1996) 179, hep-th/9604166.
49. O.J. Ganor, R. Gopakumar and S. Ramgoolam, *Higher loop effects in M(atr ix) orbifolds*, hep-th/9705188.
50. A. Hashimoto and W. Taylor, *Fluctuation Spectra of Tilted and Intersecting D-branes from the Born-Infeld Action*, hep-th/9703217.
51. M.R. Douglas, *D-branes and matrix theory in curved space*, Contribution to Strings '97, hep-th/9707228.
52. M.R. Douglas, A. Kato and H. Ooguri, *D-brane actions on Kahler manifolds*, hep-th/9708012.